

Arbitrary bending of optical solitonic beam regulated by boundary excitations in a doped resonant medium

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Bending of a shape-invariant optical beam is achieved so far along parabolic or circular curves. We propose beam bending along any preassigned curve or surface, controlled by the boundary population inversion of atoms in a doped resonant medium. The optical 2D or 3D beams generated in a nonlinear Kerr medium and transmitted through a doped medium represent exact soliton solutions of integrable systems. Arbitrary accelerating solitonic beam predicted here, should be realizable experimentally and applicable to nonlinear events in other areas like plasma or ocean waves.

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It is our long-standing dream that one day it would be possible to produce localized light beams, that could be bent into any preassigned curve with self-acceleration and shape-invariance throughout their propagation in a medium. Such beams would be able to self-bend around an obstacle, making it invisible. However, the first step in achieving a self-accelerating beam through theoretical and experimental studies has been taken quite late [1, 2], inspired by a pioneering work of Berry et al related to accelerating free quantum particles [3]. The beam, typically an Airy function solution of linear Schrödinger equation, could maintain its parabolic form over a finite distance. Subsequently, acceleration along an arbitrary curve was achieved, though at the cost of non-preservation of the shape [4]. A beautiful bending in circular path is found recently for a shape-preserving optical beam on a plane, breathing periodically in time, as a Bessel function related exact solution to the linear Maxwell equation in vacuum [5]. A parallel development has also been started in this context involving nonlinear media, in which an Airy-like beam solution is found to the nonlinear Schrödinger (NLS) equation in a Kerr medium [6, 7]. Negating the prevailing scepticism, that symmetric nonlinearities can not produce self-accelerating soliton, such a shape-preserving localized static beam solution was obtained (by replacing time t with optical axis z), oriented along a parabolic curve in a two dimensional (2D) plane, together with its trivial generalization to three dimensions (3D), which subsequently have been verified in real experiments [7, 8].

Nevertheless, in all such exciting achievements, one could obtain shape-preserving beams, which can

bend basically along two types of curves: circular or parabolic, linked to the Airy or Bessel function related solutions [1]-[8]. This situation however falls way short of our expectation of having a shape-invariant optical beam, which could bend to any desired curve, regulated by some pre-settings in a nonlinear medium. Such a stable self-accelerating beam in 2D or 3D should be able to turn around an object of arbitrary shape, turning it invisible. Our purpose here is to offer a breakthrough by proposing a scenario close to the above expectation, where a localized 2D or 3D optical beam could be accelerated in an arbitrary way, controlled by the boundary value of the population inversion of atoms in a doped resonant medium, through which the shape-preserving beam passes and hence can bend around an object without hitting it. The idea of our construction for bending of optical beams follows a fundamentally different path than those adopted so far, as briefed above, and is inspired by a series of earlier studies in the nonlinear optical communication [9]-[19], listed below.

Soliton based optical communication governed by an integrable NLS equation

$$iE_z + E_{tt} + 2|E|^2E = 0, \quad (1)$$

for the electric field $E(t, z)$ was proposed long back [10], where the group velocity dispersion of the field could be countered by its self phase modulation in a nonlinear Kerr medium, resulting to a stable shape-preserving pulse propagation in 1D [9]. Here the subscripts denote partial derivatives, with t as time and z as the coordinate along the optical axis. The NLS equation (1), being an integrable system, possesses many interesting properties like higher conserved quantities, exact N-soliton solution, integrable hierarchy and a Lax pair [11]. The $(1+1)$ dimensional NLS equation (1), a successful model in nonlinear optics can be derived from the more fundamental Maxwell equation for the electric field in a nonlinear medium [12], where the optical pulse can propa-

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gate in the form of a stable localized 1-soliton solution:

$$E = -2i \operatorname{sech} 2\xi e^{2i\theta}, \quad (2)$$

with $\xi = \eta(t - v_0 z)$, $\theta = -kt + \omega_0 z$, and vanishing electric field at the initial moment $E(t \rightarrow -\infty, z) = 0$. Note that this conventional soliton moves with a constant velocity v_0 and modulation frequency ω_0 of the enveloping wave, preserving its shape due to constant width and the wave vector, all linked to spectral parameter value $\lambda_1 = k + i\eta$. Though in the context of optical communication velocity and frequency parameters: v_0, ω_0 , represent actually their *inverses*, we continue calling them by their usual names, for convenience.

Another proposal which gained popularity for improved transmission of nonlinear optical pulses is due to self-induced transparency (SIT), produced by the coherent response of the medium to ultra short pulses [13, 14]. The governing dynamics is described by the SIT equations

$$ip_t = 2(NE - \omega_0 p), \quad iN_t = E^* p - p^* E \quad (3)$$

together with the coupling to the electric field: $iE_z = 2p$. Here $p(t, z)$ is the polarization in the resonant medium induced by the propagating electric field $E(t, z)$ and $-1 \leq N(t, z) \leq +1$, is the population inversion (PI) of the dopant atoms, ω_0 being the averaged natural frequency. The SIT equations is found also to be an integrable system associated with a Lax pair, admitting exact localized soliton solutions for all the fields E, p and N , involved in the equations with initial conditions

$$E \rightarrow 0, \quad p \rightarrow 0 \quad \text{and} \quad N \rightarrow 1, \quad \text{at} \quad t \rightarrow -\infty. \quad (4)$$

The optical pulse described by the electric field can be given in the same solitonic form (2), with changed arguments as $\xi = \eta(t - v_m z)$, $\theta = -kt + \omega_m z$, with constant velocity v_m and modulation frequency ω_m dependent on the constant spectral-related parameters k, η . A detailed account of the SIT system can be found in [15].

A subsequent important idea is to combine these two models, e.g. NLS (1) and SIT (3) equations for propagation of optical pulses in a coupled NLS-SIT system, by transmitting the optical soliton created in a Kerr medium further through an Erbium doped nonlinear medium [16, 17]. Remarkably, this coupled NLS-SIT model retains its integrability and could be associated with a Lax pair by cleverly combining those of its integrable subsystems [16–19]. The fields E, p, N admit exact soliton solutions with the optical pulse given again in the localized form (2) with arguments:

$$\xi = \eta(t - Vz), \quad \theta = -kt + \Omega z, \quad (5)$$

with a constant velocity $V = v_0 + v_m$ and modulation frequency $\Omega = \omega_0 + \omega_m$, given by a superposition of the entries from its subsystems. It is crucial to note, that the constancy of velocity V and frequency Ω is linked here to the initial conditions (4). We will see below what

happens, when the last of these conditions is relaxed. Our intention is to use the same governing equations but adopt them for the present context of optical beam bending on a 2D surface, by formally replacing the time variable t with coordinate x along the transverse direction:

$$iE_z + E_{xx} + 2|E|^2 E = 2p, \\ ip_x = 2(NE - \omega_0 p), \quad iN_x = E^* p - p^* E. \quad (6)$$

Here $p(x, z) = \nu \tilde{\nu}^*$ is the polarization in the resonant medium induced by the electric field $E(x, z)$ and $N = |\tilde{\nu}|^2 - |\nu|^2$, $-1 \leq N(x, z) \leq +1$, is the PI profile of the effective two-level dopant atoms, with normalized wave functions ν and $\tilde{\nu}$ for the ground and the excited states, respectively. We recall that the coupled NLS-SIT system (6) is an integrable system admitting stable localized optical beam in the form of a soliton solution as presented in (2) with (5) (with t replaced by x), for the the boundary conditions $E \rightarrow 0, p \rightarrow 0$ and $N \rightarrow N_0 = 1$, at $x \rightarrow \pm\infty$ (correspond to initial condition (4) for $x \rightarrow t$). Notice that, due the constant PI profile $N_0 = 1$ at the transverse boundaries, i.e. when the boundary atoms along the transverse directions are uniformly excited, one obtains soliton solution for the optical beam moving with a constant *space-velocity* V and therefore without any bending.

Arbitrary bending of 2D beam- An intriguing observation for this coupled NLS-SIT system (6) is that, instead of choosing the boundary PI as a constant: $N_0 = 1$, we can take the PI at the transverse boundaries as an arbitrary function: $N_0 = N_0(z)$ along the optical axis z , by suitably preparing the excitation profile of the dopant atoms at the boundaries in the transverse direction. This alteration in preparing of the set up brings however a significant change in the character of the optical solitonic beam propagating through the medium. Since the soliton velocity is linked directly to the boundary PI configuration $N_0(z)$, the velocity itself becomes a function $V(z)$ of z and the exact soliton solution of the system (6) undergoes a *space-acceleration*, expressed in the analytic form [19]

$$E(x, z) = -2i \operatorname{sech} 2\xi e^{2i\theta}, \quad \xi(x, z) = \eta(x - V(z)), \\ \theta(x, z) = -kx + \Omega(z). \quad (7)$$

Here the variable velocity $V(z)$ and modulation frequency $\Omega(z)$, given respectively by

$$V(z) = v_0 z + v_m f(z), \quad \Omega(z) = \omega_0 z + \omega_m f(z), \\ \text{where} \quad f(z) = \int_0^z dz' N_0(z'), \quad (8)$$

are determined by the PI profile, at the transverse boundaries:

$$N(x \rightarrow \pm\infty, z) = N_0(z). \quad (9)$$

We call $V(z)$ and $\Omega(z)$ as the velocity and the frequency for convenience, though strictly speaking their

derivatives $V'(z) = v_0 + v_m N_0(z)$ and $\Omega'(z) = \omega_0 + \omega_m N_0(z)$ should be the proper entries and be called the *space-velocity* and the *space-modulation frequency*, respectively. Consequently, the nontrivial self *space*-acceleration of the beam: $|V''(z)| = \frac{1}{R(z)}$, related to the curvature $R(z)$, should induce a bending of the optical beam. The intensity of the solitonic beam: $|E(x, z)| = 2\text{sech}2\xi$, described by (7) is peaked at $\xi = 0 = x - V(z)$ and therefore would bend along the curve $x = V(z)$ in (x, z) plane, where arbitrary function $V(z)$ is fixed by the boundary setting of the PI profile $N_0(z)$ as in (8). Note, that unlike the infinite energy Airy beam, the optical beam described here by the exact solution (7) is exponentially localized and corresponds to a finite energy solution of nonlinear equations (6). It is important to note for physical applications that, since the curve-generating function $V(z)$ is determined by the boundary PI profile $N_0(z)$, one can tune the form of the curve along which the beam would bend, by suitably designing the PI configuration of the dopant atoms at the transverse boundaries.

We present below few examples of such a bending of the peak intensity of the solitonic optical beam (7), following different curves (Fig. 1 a-d) for different choices of $V(z)$. The curve-generating arbitrary function $V(z)$ is linked to the boundary PI profile $N_0(z)$ as (8) or more directly as $N_0(z) = \frac{1}{v_m}(V'(z) - v_0)$.

i) $V(z) = z^2$, induces bending of the optical beam along a *parabolic* curve $x = z^2$ (Fig. 1a).

ii) $V(z) = (1 - \alpha z^2)^{\frac{1}{2}}$ with a constant α corresponds to an *elliptic* curve $x^2 + \alpha z^2 = 1$, along which the optical solitonic beam would bend (Fig. 1b). The curve naturally degenerates to a *circle* $x^2 + z^2 = 1$, for $\alpha = 1$.

iii) $V(z) = z^{-1}$ yielding $zx = 1$, describes a *hyperbolic* curve for the bending of the solitonic beam (Fig. 1c).

iv) For the choice $V(z) = \sin z$, the optical beam would follow a *periodic* curve $x = \sin z$ (Fig. 1d).

This result demonstrates, that in our setup the bending of a nonparaxial optical 2D beam can be achieved along any desired planer curve $x = V(z)$, by choosing suitably the arbitrary function $V(z)$, linked in turn to the boundary PI $N_0(z)$ (9), through relation (8). Therefore, the nature of the curve along which the optical beam bends, can be regulated by properly preparing the excitation profile of the dopant atoms along the boundaries in the transverse dimension x . Note, that here we obtain a self-accelerating shape-invariant stable solitonic beam as an exact solution of integrable equations, seemingly contradicting the common belief, that accelerating soliton can not exist in an integrable system, due to strict conservation laws. Resolution of this apparent paradox is that, here the acceleration of the soliton is linked to a nontrivial boundary function, which serves as an external source of energy and should also be counted in defining the conserved quantities.

Arbitrary bending of 3D beam- In spite of the success in bending of an optical beam along an arbitrary curve

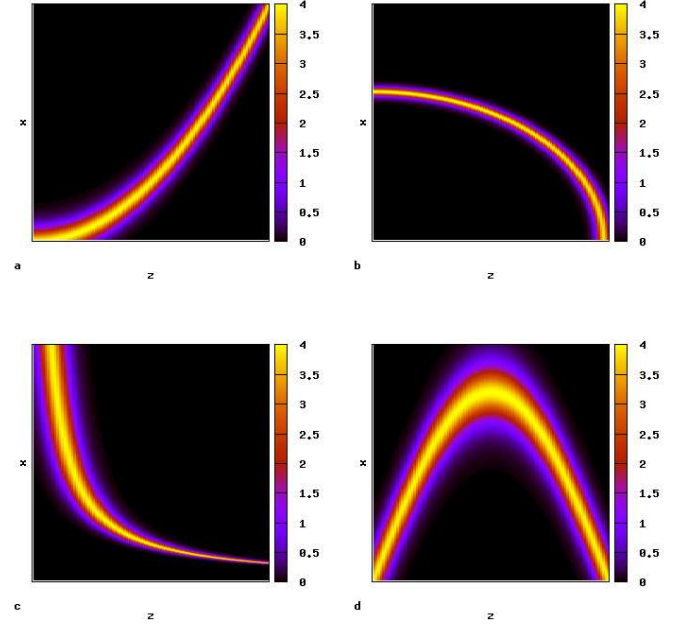


FIG. 1: Examples of the possible bending of a nonparaxial optical beam along different curves in x, z plane with optical axis $z \geq 0$ and transverse dimension x . The figure shows the intensity $|E(x, z)|$ of the solitonic beam (7) for different choices of the curve-generating function $V(z)$. The nature of the curve can be controlled by presetting the population inversion of dopant atoms along the boundaries in the transverse direction. The panels show the bending of the peak intensity of optical beams in the first quadrant of the plane (a) along a parabolic curve, (b) along an ellipse, (c) along a hyperbola and (d) along a periodic curve: $x = \sin z$.

on a 2D plane reported above, we intend to generalize this result to include another transverse dimension y , since our ultimate interest is to achieve arbitrary beam bending in a physical 3D space. This would mean, that we could create a stable shape-preserving optical beam, that would stretch along the optical axis z with an arbitrary *space* acceleration and hence could bend along an arbitrary path in a 3D space: (x, y, z) . However, it is in no way an easy task, since for this one has to find first a $(2 + 1)$ dimensional integrable generalization of the $(1 + 1)$ NLS equation (1), allowing stable soliton solutions, which subsequently has to be coupled to the SIT equations in an integrable way, in analogy with the integrable NLS-SIT system (6). Note, however that the conventional extension of the NLS equation (1) to $(2 + 1)$ dimensions with cubic nonlinearity will not serve our purpose, since it does not allow stable soliton solutions [20]. We have to look therefore for an alternative higher dimensional NLS equation, that could accommodate shape-preserving soliton solutions. Fortunately, such an extension of the NLS equation to $(2 + 1)$ dimensions has been found recently [21, 22], by replacing the standard cubic nonlinearity of the NLS equation

by a current like nonlinearity $J^t E$:

$$iE_z + E_{yt} + 2iJ^t E = 0, \quad J^t \equiv E^* E_t - E_t^* E. \quad (10)$$

This 2D NLS equation admits a stable 2D dynamical soliton solution for the electric field $E(t, y, z)$ again in the same sech form (2) which however is defined now in a $(2 + 1)$ dimensional space-time: (y, z, t) with its arguments given as

$$\xi = \eta(t - v_1 y - v_2 z), \quad \theta = -kt + \omega_1 y + \omega_2 z, \quad (11)$$

where the velocity components (v_1, v_2) as well as modulation frequencies (ω_1, ω_2) are constants and depend only on the real and imaginary parts of a fixed spectral parameter $\lambda_1 = k + i\eta$. The higher dimensional NLS equation (10) is associated with a Lax pair $(U_2(\lambda), V_3(\lambda))$, containing higher powers in spectral parameter λ , exhibiting thus an important signature of its integrability [21, 22]. For confirming the importance of this 2D NLS equation in the context of nonlinear optics, it can be shown that Eq. (10), similar to the well known 1D NLS equation, is also derivable from the Maxwell equation for the electric field in a medium with nonlinear polarization [12]. In nonlinear optics the soliton like solution (11) for the electric field $E(t, y, z)$, would represent a shape-preserving stable optical beam transmitted through a 2D nonlinear Kerr medium, which can move only with a constant velocity $\mathbf{v} = (v_1, v_2)$, as seen from $\xi(t, y, z)$.

However, since our aim is to achieve a stable 3D optical beam with bending, we have to create an accelerating soliton solution in place of the constant-velocity soliton (11) and therefore, in analogy with the coupled NLS-SIT system described above, we propose to couple the $(2 + 1)$ -dimensional NLS equation (10) with the SIT equations (3) (by replacing $t \rightarrow x$, suitable in our context) to have a 2DNLS-SIT system as

$$\begin{aligned} iE_z + E_{yx} + 2iE(E^* E_x - E E_x^*) &= 2p, \\ ip_x &= 2(NE - \omega_0 p), \quad iN_x = E^* p - p^* E, \end{aligned} \quad (12)$$

where $p(x, y, z)$ is the polarization in the 3D resonant medium induced by the electric field $E(x, y, z)$ and $-1 \leq N(x, y, z) \leq +1$ is the PI profile of the dopant atoms as in (6), though generalized here for the 3D space. In the set of equations (12) we have replaced time variable t with the transverse dimension x for describing nonparaxial light beams along optical axis z with two transverse dimensions x, y . We find importantly, that the coupled set of 2DNLS-SIT equations (12) is an integrable system associated with a deformation of the 2DNLS Lax pair as $(U_2(\lambda), V_3(\lambda) + V_{-1}(\lambda))$, with an additional term $V_{-1}(\lambda, N, p, p^*)$, containing the SIT fields: N, p, p^* and an inverse power of the spectral parameter: λ^{-1} . Recall that, we have presented above a similar NLS-SIT system, but in lower dimensions with the details given in [18, 19]. For localized soliton solutions we are interested in, the electric and the polarization fields

should vanish at the boundaries $x \rightarrow \pm\infty$ in the transverse directions: $E(\pm\infty, y, z) \rightarrow 0$ and $p(\pm\infty, y, z) \rightarrow 0$, while the PI with nontrivial boundary values: $N(x \rightarrow \pm\infty, y, z) = N_0$, where $-1 \leq N_0(y, z) \leq +1$, could be, in general, an arbitrary function of y and z . We will see that, $N_0(y, z)$ arranged at the transverse boundaries of x would be responsible for the arbitrary *space* acceleration of the solitonic beam and hence for its bending along an arbitrary surface embedded in a 3D space.

It is remarkable that, the set of higher dimensional integrable equations (12), similar to the NLS-SIT system [18, 19], allow a localized stable soliton solution as

$$\begin{aligned} E(x, y, z) &= -2i \operatorname{sech} 2\xi e^{2i\theta}, \\ \xi &= \eta(x - v_1 y - v_2(y, z)), \quad \theta = -kx + \omega_1 y + \omega_2(y, z), \end{aligned} \quad (13)$$

together with the soliton solutions for the other fields $p(x, y, z)$ and $N(x, y, z)$. Note that (13) is a 3D generalization of (7) and a deformation of (11), where the velocity $v_2(y, z)$ and the modulation frequency $\omega_2(y, z)$ components along the z -axis have been changed to include y, z -dependence due to more general coordinate dependant choice for the boundary PI $N_0(y, z)$, as evident from their explicit form

$$\begin{aligned} v_2(y, z) &= v_2 z + v_{3m} f(y, z), \quad \omega_2(y, z) = \omega_2 z + \omega_{3m} f(y, z) \\ \text{where} \quad f(y, z) &= \int_0^z dz' N_0(y, z') \end{aligned} \quad (14)$$

where the constant parameters v_{3m}, ω_{3m} are determined through a fixed spectral parameter $\lambda_1 = k + i\eta$. Since the intensity of the exponentially localized solution (13) is peaked over the surface $\xi(x, y, z) = 0$, the localized optical beam described by this solution would *space*-accelerate as $\partial_{zz}^2 v_2(y, z)$ along both z and y directions and consequently, would self-bend over an arbitrarily curved surface $x = v_1 y + v_2(y, z)$. The form of this 3D surface is determined by an arbitrary function $v_2(y, z)$, which in turn is linked to $N_0(y, z)$ as in (14). It is therefore possible to bend a 3D optical beam in our arrangement over any preassigned manifold by regulating the setting of the PI function $N_0(y, z)$, prepared at the boundary in the transverse direction $x \rightarrow \pm\infty$. We demonstrate in Fig. 2 an example, where the curved surface along which the 3D beam can bend is described by $x = y^2 + z^2$, representing a conical surface, spreading out with circular projections of increasing radius along the transverse direction $x > 0$ as its central axis, with its vertex at $x = 0$. Such an optical beam can turn around a 3D object placed inside the hollow of the cone, making it invisible. In this example, the surface-generating function $v_2(y, z) = y^2 + z^2 - v_1 y - v_2 z$, is linked to the boundary PI through the relation: $N_0(y, z) = \frac{1}{v_m} (\partial_z v_2(y, z) - v_2)$. The other choices of function $v_2(y, z)$ and hence for the boundary PI $N_0(y, z)$ would generate other forms of the surface in the 3D space, over which the intensity peak of an optical beam would bend.

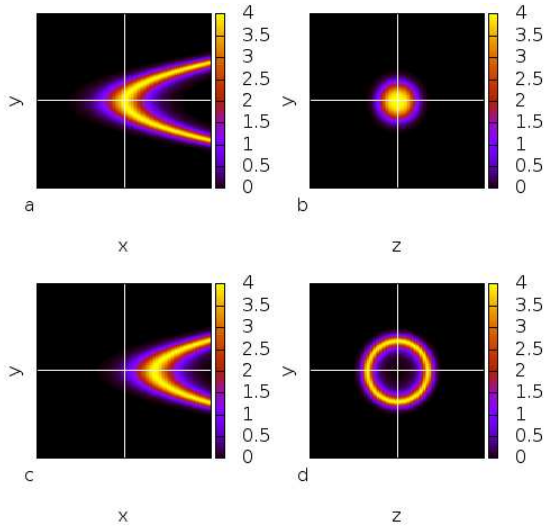


FIG. 2: Conical surface over which a 3D optical solitonic beam can bend turning invisible any object placed inside the cone. Figure shows the beam intensity $|E(x, y, z)| = \text{sech } \xi(x, y, z)$ for a particular choice of function $v_2(y, z)$ (see (13)), where z is the optical axis and x, y are transverse dimensions. The panels show for clarity different cross-sections of the conical surface (for physical interpretation consider only the positive quadrants), a) with major vertical section at $z = 0.0$, b) the vertex of the cone at $x = 0.0$, c) another vertical section of the surface at $z = 1.3$, d) a horizontal section showing a circular projection at $x = 3.0$.

Based on our findings here we conclude therefore that, an optical beam created in a nonlinear Kerr

medium as a stable localized soliton, can acquire an arbitrary acceleration while transmitted through an Erbium doped medium, in spite of being an integrable system, due to a path-dependant boundary condition. This causes therefore bending of an optical beam along an arbitrary curve on a 2D plane or over a curved surface in the 3D space, which can be linked to a nontrivial population inversion of the effective two-level atoms of the doped medium at the boundaries of the transverse direction. The form of the curve or the surface for the bending of the beam therefore, can be controlled by carefully exciting the atoms of the doped medium by pre-pumping through an external laser. This pumping of external energy, in fact, is responsible for the accelerated motion of the optical solitonic beam. This arrangement should work both in two and three dimensional spaces by coupling the corresponding integrable NLS equation with the SIT equations, which would yield exact finite energy localized beam solutions with self acceleration, inducing arbitrary bending. We hope that this possibility, based on the idea used already in nonlinear optical communication, for bending arbitrarily a 2D or 3D optical beam beyond parabolic or a circular curve, would be feasible for experimental verification, opening up new vistas of practical applications. Moreover, due to the importance of the NLS equation and related soliton solutions in nonlinear phenomena in diverse areas in physics like plasma, ocean waves, optical communications etc. the possibility of generating accelerating solitons in a related nonlinear system should be of significant applicable interest.

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Supplementary material to *Arbitrary bending of an optical soliton beam....by A. Kundu et al*

I. DERIVATION OF INTEGRABLE 1D NLS AND 2D NLS EQUATIONS FROM THE MAXWELL EQUATION IN A MEDIUM

A. Maxwell equation for the electric field in a nonlinear medium

Since the propagation of light inside an optical medium is guided by Maxwell equation in the absence of free charges:

$$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}, \quad (15)$$

we intend to derive both the one-dimensional (1D) as well as the two-dimensional (2D) nonlinear Schrödinger (NLS) equation (presented in the main text as Eq. (1) and (10)) from the Maxwell equation, to confirm their relevance in the context of the nonlinear optics. We closely follow the procedure for deriving the equations like 1D NLS and the derivative NLS as detailed in [1] and slightly extend it to accommodate the derivation of the 2D NLS.

Using further $\nabla \times \nabla \times \mathbf{E} \equiv \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E}$, due to $\nabla \cdot \mathbf{E} = 0$, and taking Fourier transform

$$\tilde{\mathbf{E}}(\mathbf{r}, \omega - \omega_0) = \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r}, t) \exp(i(\omega - \omega_0)t) dt \quad (16)$$

equation (15) reduces to

$$\nabla^2 \tilde{\mathbf{E}}(\mathbf{r}, \omega - \omega_0) + k_0^2 \tilde{\mathbf{E}}(\mathbf{r}, \omega - \omega_0) = \mu_0 \frac{\partial^2 \tilde{\mathbf{P}}}{\partial t^2}, \quad (17)$$

where the polarization is given by

$$\mathbf{P} = \mathbf{P}_L + \mathbf{P}_{NL}, \quad (18)$$

with the linear polarization

$$\mathbf{P}_L = \frac{\epsilon_0}{2\pi} \int_{-\infty}^{\infty} \chi^{(1)}(\omega) \tilde{\mathbf{E}}(\mathbf{r}, \omega - \omega_0) \exp(-i(\omega - \omega_0)t) d\omega \quad (19)$$

and nonlinear polarization

$$\mathbf{P}_{NL} = \epsilon_0 \chi^{(3)} \mathbf{E}(\mathbf{r}, t) \int_{-\infty}^t R(t - t_1) |E(\mathbf{r}, t_1)|^2 dt_1 \quad (20)$$

with $R(t)$ being the nonlinear response function normalized as $\int_{-\infty}^{\infty} R(t) dt = 1$

Taking the Fourier transform also with respect to y we can obtain

$$\tilde{\mathbf{E}}(\rho, \Omega - \Omega_0, \omega - \omega_0) = \int_{-\infty}^{\infty} \tilde{\mathbf{E}}(\rho, y, \omega - \omega_0) e^{i(\Omega - \Omega_0)y} dy \quad (21)$$

where $\rho = (x, z)$. By adopting the separation of variable method and the slowly varying envelope approximation [1], we obtain

$$\tilde{E}(\rho, \Omega - \Omega_0, \omega - \omega_0) = F(x) \tilde{A}(z, \Omega - \Omega_0, \omega - \omega_0) e^{i\beta_0 z} \quad (22)$$

Following the details given in [1] one obtains further from Eq. (17) and (22) the set of equations

$$F_{xx} + [\epsilon(\omega) k_0^2 - \tilde{\beta}^2] F = 0 \quad (23)$$

$$2i\beta_0 \tilde{A}_z + [\tilde{\beta}^2 - \beta_0^2] \tilde{A} = \mu_0 \frac{\partial^2 \tilde{P}_{NL}}{\partial t^2} \quad (24)$$

where $\epsilon(\omega) = 1 + \chi^{(1)}(\omega)$, $\tilde{\beta} = \tilde{\beta}(\Omega, \omega)$ and the scalar form of polarization is used with slowly varying optical field. Using (20) therefore, after inverse Fourier transform in t and y , Eq. (24) can be written as

$$\begin{aligned} & \frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \beta_{10} \frac{\partial A}{\partial t} + \frac{i\beta_{20}}{2} \frac{\partial^2 A}{\partial t^2} \\ & + \beta_{01} \frac{\partial A}{\partial y} + \frac{i\beta_{02}}{2} \frac{\partial^2 A}{\partial y^2} + i\beta_{11} \frac{\partial^2 A}{\partial y \partial t} \\ & = i\gamma \left(1 + \frac{i}{\omega_0} \frac{\partial}{\partial t} \right) \left(A(t, y, z) \int_{-\infty}^{\infty} R(t') |A(t - t', y, z)|^2 dt' \right) \end{aligned} \quad (25)$$

where $\beta_{mn} = \frac{\partial^{m+n}\beta}{\partial \omega^m \partial \Omega^n}$ are the dispersion coefficients and $T_R = \int_{-\infty}^{\infty} t R(t) dt$ is the first moment of the nonlinear response function. Note that, using $|A(t - t', y, z)|^2 = |A(t, y, z)|^2 - t' \partial_t |A(t, y, z)|^2$ for the slowly varying pulse envelope we can rewrite Eq. (25) as

$$\begin{aligned} & \frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \beta_{10} \frac{\partial A}{\partial t} + \frac{i\beta_{20}}{2} \frac{\partial^2 A}{\partial t^2} \\ & + \beta_{01} \frac{\partial A}{\partial y} + \frac{i\beta_{02}}{2} \frac{\partial^2 A}{\partial y^2} + i\beta_{11} \frac{\partial^2 A}{\partial y \partial t} \\ & = i\gamma \left(|A|^2 A + \frac{i}{\omega_0} \frac{\partial}{\partial t} (|A|^2 A) - T_R (A \frac{\partial}{\partial t} |A|^2) \right), \end{aligned} \quad (26)$$

where the RHS is simplified and a second order term involving the ratio $\frac{T_R}{\omega_0}$ has been neglected due to its smallness, as done in [1].

From the general form of equation (26), which is derived here from the Maxwell equation (15) we can derive now 1D and 2D NLS equations at different limits, as we show below.

B. Derivation of 1D NLS equation

Considering the propagation of the field $A(t, z)$ only along the z -direction, we can drop all terms with $\frac{\partial}{\partial y}$ in (26) due to the independence of the field on the transverse dimension. Taking further a loss-free system $\alpha = 0$

with pulses of width $T_0 > 5ps$, resulting to very small parameters: $\frac{1}{\omega_0 T_0} \rightarrow 0$ and $\frac{T_R}{T_0} \rightarrow 0$, we can neglect last two terms in the RHS of Eq. (26) to reduce it to

$$\frac{\partial A}{\partial z} + \beta_{10} \frac{\partial A}{\partial t} + \frac{i\beta_{20}}{2} \frac{\partial^2 A}{\partial t^2} = i\gamma|A|^2 A, \quad (27)$$

which shifting to a frame moving with the pulse (retarded frame) and absorbing the coefficients by scaling is transformed to the dimensionless form for the 1 + 1-dimensional NLS equation (presented in the main text as Eq. (1)).

C. Derivation of 2D NLS equation

Consider now more general case, when the field $A(t, y, z)$, propagating along the optical axis z depends additionally on the transverse direction y , with a loss-free system $\alpha = 0$, in the complementary case of ultra-local optical pulse with $T_0 < 1ps$, together with small nonlinear coupling: $\gamma \rightarrow 0$, where we can neglect now the first term, but not the last two terms in the RHS of Eq. (26). Assuming further vanishing values of the dis-

person coefficients: $\beta_{i0} \rightarrow 0$, $i = 1, 2$, $\beta_{02} \rightarrow 0$, and going to a retarded frame of reference $t \rightarrow t - \beta_{1,0}z$, the general equation (26) can be reduced to a simpler form

$$\frac{\partial A}{\partial z} + i\beta_{11} \frac{\partial^2 A}{\partial y \partial t} + \gamma A \left(\left(\frac{2}{\omega_0} + iT_R \right) A^* A_t + \left(\frac{1}{\omega_0} + iT_R \right) A_t^* A \right) = 0 \quad (28)$$

which by imposing an additional condition $T_R = \frac{3i}{2\omega_0}$, reduces further to

$$i \frac{\partial A}{\partial z} - \beta_{11} \frac{\partial^2 A}{\partial y \partial t} + i \frac{\gamma}{2\omega_0} A \left(A^* \frac{\partial A}{\partial t} - A \frac{\partial A^*}{\partial t} \right) = 0 \quad (29)$$

By absorbing the coefficients through scaling of the variables, we get finally the required dimensionless form of the 2D NLS equation (presented in the main text as Eq. (10)) and coupling it to the self induced transparency equations results to the *NLS-SIT* system.

This completes the derivation of both 1+1 and 1+2 dimensional nonlinear Schrödinger (NLS) equation from the Maxwell equation.